Soluciones

1. $\begin{vmatrix} a & b \\ b & a \end{vmatrix} = a^2 - b^2 = (a+b)(a-b)$

$\begin{vmatrix} \text{sen}(\alpha) & \cos(\alpha) \\ \text{sen}(\beta) & \cos(\beta) \end{vmatrix} = \text{sen}(\alpha)\cos(\beta) - \cos(\alpha)\text{sen}(\beta) =$

$= \text{sen}(\alpha - \beta)$

$\begin{vmatrix} \log(4) & \log(4) \\ \log(2) & \log(20) \end{vmatrix} = \log(4)\cdot\log(20) - \log(4)\cdot\log(2) =$

$= \log\dfrac{4\cdot 20}{4\cdot 2} = \log(10) = 1$

2. $\begin{vmatrix} 1 & -1 & 0 \\ 2 & 1 & 1 \\ 3 & 0 & -1 \end{vmatrix} = -1 - 3 - 2 = -6$

$\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix} = a^3 + b^3 + c^3 - (cab + bca + abc) =$

$= a^3 + b^3 + c^3 - 3abc$

3. a) $\begin{vmatrix} ab & 2ac & 3a^2 \\ 2b^2 & bc & ab \\ 2bc & c^2 & 2ac \end{vmatrix} = abc\begin{vmatrix} b & 2c & 3a \\ 2b & c & a \\ 2b & c & 2a \end{vmatrix} =$

$= a^2b^2c^2\begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = -3a^2b^2c^2$

b) $\begin{vmatrix} a^2 & a & bc \\ b^2 & b & ca \\ c^2 & c & ab \end{vmatrix} \overset{\substack{a\cdot F_1 \\ b\cdot F_2 \\ c\cdot F_3}}{=} \dfrac{1}{abc}\begin{vmatrix} a^3 & a^2 & abc \\ b^3 & b^2 & bca \\ c^3 & c^2 & cab \end{vmatrix} =$

$= \dfrac{abc}{abc}\begin{vmatrix} a^3 & a^2 & 1 \\ b^3 & b^2 & 1 \\ c^3 & c^2 & 1 \end{vmatrix} = \begin{vmatrix} a^3 & a^2 & 1 \\ b^3 & b^2 & 1 \\ c^3 & c^2 & 1 \end{vmatrix}$

4. a) $\begin{vmatrix} 6 & 0 \\ 2 & 1 \end{vmatrix} = 6 \neq 0 \Rightarrow \text{rg}(A) \geqslant 2$

$\begin{vmatrix} 6 & 0 & -1 \\ 2 & 1 & -3 \\ 1 & 4 & -2 \end{vmatrix} = -12 - 8 - (-1 - 72) =$

$= 53 \neq 0 \Rightarrow \text{rg}(A) = 3$

b) $\begin{vmatrix} -2 & 3 \\ 3 & 2 \end{vmatrix} = -13 \neq 0 \Rightarrow \text{rg}(A) \geqslant 2$

$\begin{vmatrix} -2 & 3 & 1 \\ 3 & 2 & -1 \\ 4 & -1 & 0 \end{vmatrix} = -21 \Rightarrow \text{rg}(A) \geqslant 3$

$\begin{vmatrix} -2 & 3 & 1 & 0 \\ 3 & 2 & -1 & 2 \\ 4 & -1 & 0 & 5 \\ 3 & 7 & 1 & 7 \end{vmatrix} = 0, \begin{vmatrix} -2 & 3 & 1 & 5 \\ 3 & 2 & -1 & 2 \\ 4 & -1 & 0 & 3 \\ 3 & 7 & 1 & 15 \end{vmatrix} = 0 \Rightarrow \text{rg}(A) = 3$

5. a) $\begin{vmatrix} 1 & 1 & 1 & 1 \\ -1 & x & 1 & 1 \\ -1 & -1 & x & 1 \\ -1 & -1 & -1 & x \end{vmatrix} \overset{\substack{F_2+F_1 \\ F_3+F_1 \\ F_4+F_1}}{=} \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & x+1 & 2 & 2 \\ 0 & 0 & x+1 & 2 \\ 0 & 0 & 0 & x+1 \end{vmatrix} =$

$= (x+1)^3$

b) Si $x \neq -1$, $|A| \neq 0 \Rightarrow \text{rg}(A) = 4$.

Si $x = -1$, $|A| = 0 \Rightarrow \text{rg}(A) < 4$.

$\begin{vmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ -1 & -1 & 1 \end{vmatrix} = 2 \neq 0 \Rightarrow \text{rg}(A) = 3$

6. $|A| = \begin{vmatrix} 1 & 0 & -1 \\ 0 & a & 3 \\ 4 & 1 & -a \end{vmatrix} = -a^2 + 4a - 3$

a) Si $a = 1$ o $a = 3$, $|A| = 0$; luego A no es regular; en caso contrario sí lo es, y, por tanto, tiene inversa.

b) Si $a = 2$, $|A| = 1$

$A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 2 & 3 \\ 4 & 1 & -2 \end{pmatrix} \Rightarrow A^{-1} = \begin{pmatrix} -7 & -1 & 2 \\ 12 & 2 & -3 \\ -8 & -1 & 2 \end{pmatrix}$

7. $|A| = \begin{vmatrix} 1 & -1 & 0 \\ n & n+1 & n \\ 2n & 2n+1 & 2n+1 \end{vmatrix} = 3n + 1$

Si $n \neq -\dfrac{1}{3}$, $|A| \neq 0 \Rightarrow$ la matriz es regular.

Si $n = 0$, $|A| = 1$, y se tiene:

$A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \Rightarrow A^{-1} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$

8. $AX - B + C = 0 \Rightarrow AX = B - C \Rightarrow X = A^{-1}(B - C)$

$X = \begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix}^{-1}\left[\begin{pmatrix} 14 & 8 & 2 \\ 1 & 3 & -5 \end{pmatrix} - \begin{pmatrix} 1 & 3 & -6 \\ 3 & 0 & 2 \end{pmatrix}\right] =$

$= \begin{pmatrix} \frac{1}{7} & \frac{3}{7} \\ \frac{2}{7} & -\frac{1}{7} \end{pmatrix}\begin{pmatrix} 13 & 5 & 8 \\ -2 & 3 & -7 \end{pmatrix} = \begin{pmatrix} 1 & 2 & -\frac{13}{7} \\ 4 & 1 & \frac{23}{7} \end{pmatrix}$

9. $\begin{cases} 3X + Y = A \\ X - 2Y = 2B \end{cases} \overset{\substack{2E_1+E_2 \\ E_1-3E_2}}{\Rightarrow} \begin{cases} 7X = 2A + 2B \\ 7Y = A - 6B \end{cases} \Rightarrow \begin{cases} X = \frac{2}{7}(A+B) \\ Y = \frac{1}{7}(A-6B) \end{cases}$

$X = \dfrac{2}{7}\left[\begin{pmatrix} 2 & -1 \\ 1 & -3 \end{pmatrix} + \begin{pmatrix} 0 & -3 \\ 1 & -1 \end{pmatrix}\right] = \begin{pmatrix} \frac{4}{7} & -\frac{8}{7} \\ \frac{4}{7} & -\frac{8}{7} \end{pmatrix}$

$Y = \dfrac{1}{7}\left[\begin{pmatrix} 2 & -1 \\ 1 & -3 \end{pmatrix} - 6\begin{pmatrix} 0 & -3 \\ 1 & -1 \end{pmatrix}\right] = \begin{pmatrix} \frac{2}{7} & \frac{17}{7} \\ -\frac{5}{7} & \frac{3}{7} \end{pmatrix}$