# Soluciones propuesta A

**1.** a) $D(f)$ = **R** – {–3, 0}. Es continua en todo $D$.

$f'(x) = \dfrac{2(3-x)^2}{x^3(x+3)^2} \Rightarrow$ puntos singulares y críticos: $x = \pm\sqrt{3}$

b) $D(f) = \{x \in \mathbf{R} \,/\, \text{sen}\, 2x > 0\} = \left(k\pi,\ k\pi + \dfrac{\pi}{2}\right), k \in \mathbf{Z}$

Continua en $D(f)$. $g'(x) = 2\,\text{cotg}(2x) \Rightarrow$ ptos. singulares y críticos: $x = \dfrac{\pi}{4}(2k+1)$, $k \in \mathbf{Z}$

**2.** a) Eje $X$: $\left(-\dfrac{\pi}{4} + k\pi, 0\right)$, $k \in \mathbf{Z}$. Eje $Y$: (0, 1)

$f'(x) > 0$ si $x \in \left(-\dfrac{\pi}{4} + k\pi,\ \dfrac{\pi}{2} + k\pi\right)$, $k \in \mathbf{Z}$

b) Eje $X$: (ln 2, 0). Eje $Y$: $\left(0, \dfrac{1}{3}\right)$

$g(x) > 0$ si $x \in (2, +\infty)$

**3.** a) $f(x) = \text{sen}\, 3x = \text{sen}(3x + 2\pi) = \text{sen}\left(3\left(x + \dfrac{2\pi}{3}\right)\right)$

Período $T = \dfrac{2\pi}{3}$

b) $g(x) = 4\cos 2x + \text{sen}\, 3x = g_1(x) + g_2(x)$

$T_1 = \pi,\ T_2 = \dfrac{2\pi}{3} \Rightarrow T = \text{m.c.m.}(T_1, T_2) = 2\pi$

**4.** a) $f(-x) = \dfrac{-x^3 + 2x^2}{x^2 - 9} \neq \pm f(x) \Rightarrow$ Ni par ni impar

b) $g(-x) = \dfrac{e^{-x} + e^x}{-x\cos(-x)} = -g(x) \Rightarrow$ Impar

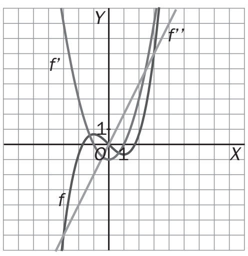
c) $h(-x) = \ln((-x)^2 - 1) = h(x) \Rightarrow$ Par

**5.** a) $f(x) = \dfrac{2x^3}{1+x^2} = 2x - \dfrac{2x}{x^2+1}$. Solo tiene la asíntota oblicua $y = 2x$.

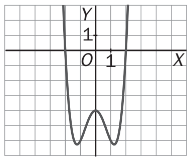
b) Vertical en $x = 0$, al ser $\lim\limits_{x\to 0} g(x) = \pm\infty$.

Horizontales: $\begin{cases} \lim\limits_{x\to -\infty} \dfrac{4+e^x}{1-e^x} = 4 \Rightarrow y = 4 \\ \lim\limits_{x\to +\infty} \dfrac{4+e^x}{1-e^x} = -1 \Rightarrow y = -1 \end{cases}$

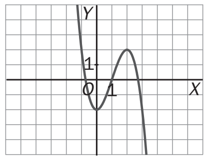
**6.**




**7.** a)



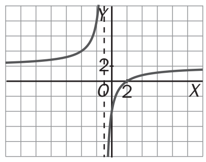
b)



**8.** a) $D(f)$ = **R** – {–1}

Cortes: (2, 0), (0, –4)

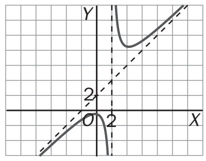
AV: $x$ = –1, AH: $y$ = 2



b) $D(g)$ = **R** – {2}

Cortes: $\left(0, \dfrac{1}{2}\right)$

AV: $x$ = 2, AO: $y = x+2$

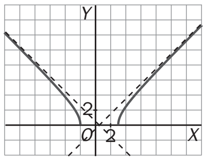


c) $D(f) = (-\infty, -2) \cup (3, +\infty)$

Cortes: (–2, 0), (3, 0)

AO: $y = x - \dfrac{1}{2}$, si $x \to +\infty$

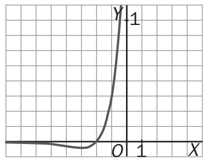
$y = \dfrac{1}{2} - x$, si $x \to -\infty$



**9.** a) $D(f)$ = **R**

Cortes: (–2, 0), (0, 2)

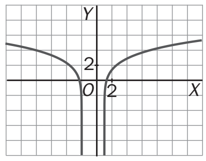
AH: $y$ = 0, si $x \to -\infty$



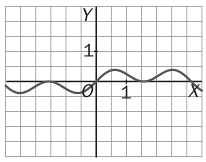
b) $D(g) = (-\infty, -2) \cup (1, +\infty)$

Cortes: no hay

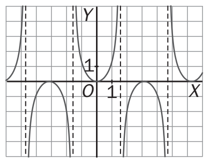
AV: $x$ = –2, $x$ = 1



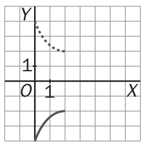
**10.** a)



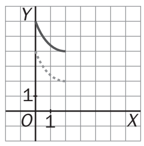
b)



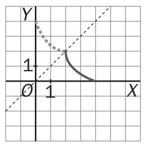
**11.** a)



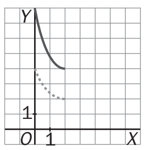
c)



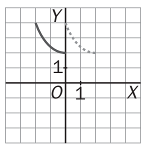
e)



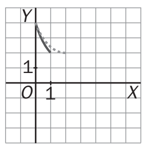
b)



d)



f)



# Soluciones propuesta B

**1.** a) $D(f)=(-\infty,-2)\cup[3,+\infty)$. Continua en $D(f)-\{3\}$. $f'(x)=\dfrac{5}{2}(x+2)^{-\frac{3}{2}}(x-3)^{-\frac{1}{2}}\neq 0\Rightarrow$ no hay puntos singulares. Tampoco hay críticos.

b) $D(g)=[3,+\infty)$. Continua en $D(g)-\{3\}$.

$g'(x)=\dfrac{5}{2}(x+2)^{-\frac{3}{2}}(x-3)^{-\frac{1}{2}}\neq 0\Rightarrow$ no hay puntos singulares. Tampoco hay críticos.

**2.** a) Eje $X$: $\left(1+e^2,\,0\right)$. Eje $Y$: no hay.

$f'(x)>0$ si $x\in\left(1,\,1+e^2\right)$

b) Eje $X$: (–2, 0). Eje $Y$: (0, 2)

$g(x)>0$ si $x\in(2,+\infty)$

**3.** a) $f(x)=\text{sen}^2\,x+\cos x=f_1(x)+f_2(x)$

$T_1=\pi,\ T_2=2\pi\Rightarrow T=\text{m.c.m}(T_1,\,T_2)=2\pi$

b) $T$ = 2 porque

$$h(x+2)=\left(\frac{x+2}{2}-\text{Ent}\left(\frac{x+2}{2}\right)\right)=$$

$$=\frac{x}{2}+1-\text{Ent}\left(\frac{x}{2}+1\right)=\frac{x}{2}-\text{Ent}\left(\frac{x}{2}\right)=h(x)$$

**4.** a) $f(-x)=\dfrac{\ln\left|x^2-5\right|}{-x}=-f(x)\Rightarrow$ Impar

b) $h(-x)=\dfrac{-x^3-2x}{x^2-1}=-h(x)\Rightarrow$ Impar

c) $h(-x)=\text{tg}((-x)^2+1)=h(x)\Rightarrow$ Par

**5.** a) Vertical en $x$ = 0, ya que

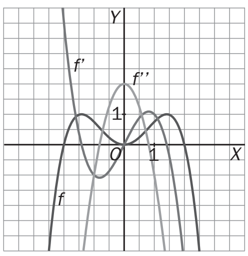
$$\lim_{x\to0^+}\left(x-\frac{2}{x}-3\ln x\right)=\lim_{x\to0^+}\left(\frac{-2-3x\ln x}{x}\right)=-\infty$$

b) $m_+=\displaystyle\lim_{x\to+\infty}\frac{\sqrt{x^2-x}}{x}=1,\ m_-=\lim_{x\to-\infty}\frac{\sqrt{x^2-x}}{x}=-1$

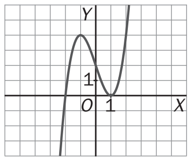
$$n_\pm=\lim_{x\to\pm\infty}\left(\sqrt{x^2-x}\mp x\right)=\lim_{x\to\pm\infty}\frac{\mp x}{\sqrt{x^2-x}+x}=\mp\frac{1}{2}$$

Asíntotas oblicuas: $\begin{cases} y=x-\dfrac{1}{2} & \text{si } x\to+\infty \\ y=-x+\dfrac{1}{2} & \text{si } x\to-\infty \end{cases}$

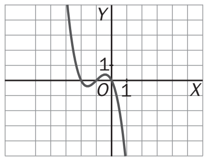
**6.**




**7.** a)



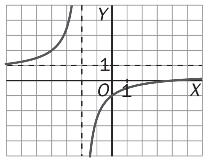
b)



**8.** a) $D(f)$ = **R** – {–2}

Cortes: (4, 0), (0, –1)

AV: $x$ = –2, AH: $y=\dfrac{1}{2}$

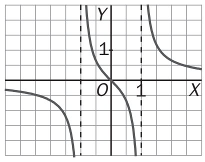


b) $D(g)$ = **R** – {–1, 1}

Cortes: (0, 0)

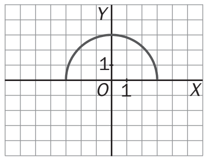
AV: $x$ = –1, $x$ = 1

AH: $y$ = 0



c) $D(h)$ = [–3, 3]

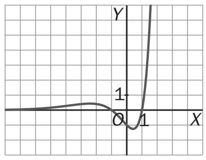
Cortes: (–3, 0), (3, 0), (0, 3)



**9.** a) $D(f)$ = **R**

Cortes: (–1, 0), (1, 0), (0, –1)

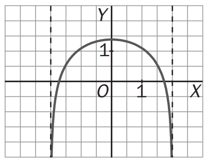
AH: $y$ = 0, si $x\to-\infty$



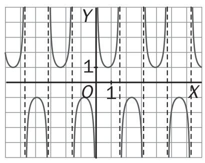
b) $D(g)$ = (–2, 2)

Cortes: (0, ln 4)

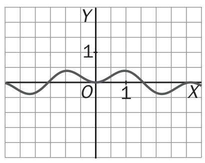
AV: $x$ = –2, $x$ = 2



**10.** a)

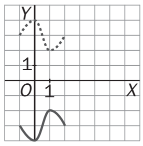


b)

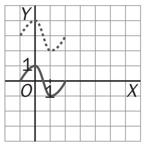


**11.**

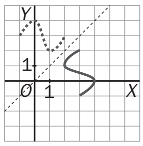
a)



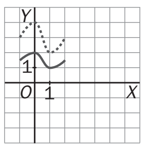
c)



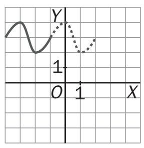
e)



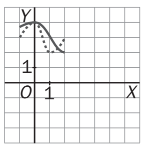
b)



d)



f)



En el apartado e se representa la correspondencia inversa de $f$ al no existir $f^{-1}$.