Soluciones propuesta A

1. $F(x)=\int\left(15x^2-2\right)dx=5x^3-2x+C$

$F(-2)=5\cdot(-2)^3-2\cdot(-2)+C=-23\Rightarrow$

$\Rightarrow C=13\Rightarrow F(x)=5x^3-2x+13$

2. $f''(x)=6x+6\Rightarrow f'(x)=3x^2+6x+C$. Como hay un máximo relativo en $x=-3$, entonces $f'(-3)=0\ \Rightarrow C=-9$ y es $f'(x)=3x^2+6x-9$.

$f(x)=x^3+3x^2-9x+K$ y como $f(-3)=17\Rightarrow$

$\Rightarrow K=-10\Rightarrow f(x)=x^3+3x^2-9x-10$.

Inflexión: $6x+6=0\Rightarrow x=-1\Rightarrow I(-1,1)$

$f'(x)=3x^2+6x-9$

$\Rightarrow\begin{cases}x_1=-3,\ \text{máximo}(-3,\,17)\\ x_2=1,\ \text{mínimo}(1,-15)\end{cases}$

Punto de corte con el eje *Y*: *P* (0, –10)

3. $v=\int a\,dt=\int -10\,dt=-10t+C$,

$v(2)=15\Rightarrow C=35$

$s=\int v dt=\int(-10t+35)dt=-5t^2+35t+k$

$s(2)=25\Rightarrow -20+70+K=25\Rightarrow K=-25$

a) $v(t)=-10t+35$ m/s

b) $v_0=v(0)=35$ m/s

c) $s(t)=-5t^2+35t-25$ m

d) $s_0=s(0)=-25$ m

e) $s(4)=35$ m, $v(4)=-5$ m/s

4. a) $I=6\ln|x+1|+C$

b) $I=-\dfrac{1}{x}+\ln|x|+x+\dfrac{x^2}{2}+\dfrac{x^3}{3}+K$

c) $I=\ln|3+\operatorname{sen}x|+K$

d) $I=\dfrac{1}{2}\int 2x\,e^{x^2+2}dx=\dfrac{1}{2}e^{x_2+2}+C$

e) $I=\int\dfrac{1}{x}\cdot\dfrac{1}{\ln x}dx=\ln|\ln x|+C$

f) $I=\sqrt{x^2+1^2}+C$

5. a) $I=\operatorname{sen}x\cos x+\int\operatorname{sen}^2x\,dx=\operatorname{sen}x\cos x+$

$+\int(1-\cos^2x)dx=\operatorname{sen}x\cos x+x-I\Rightarrow$

$\Rightarrow I=\dfrac{x+\operatorname{sen}x\cos x}{2}+C$

b) $I=x(\ln x)^3-\int 3(\ln x)^2dx$

$=x(\ln x)^3-3x(\ln x)^2+\int 6\ln x\,dx=$

$=x\left[(\ln x)^3-3(\ln x)^2+6\ln x+6\right]+C$

c) $I=-x\operatorname{cotg}x+\int\operatorname{cotg}x\,dx=$

$=-x\operatorname{cotg}x+\ln|\operatorname{sen}x|+C$

6. a) $I=\int\left(\dfrac{1}{x-2}-\dfrac{1}{x+2}\right)dx=\ln\left|\dfrac{x-2}{x+2}\right|+C$

b) $I=\int\left(2-\dfrac{5}{x+2}\right)dx=2x-5\ln|x+2|+C$

c) $I=\int\dfrac{2x+2}{(x-1)^2}dx=\int\dfrac{2x-2+4}{(x-1)^2}dx=$

$=\int\left(\dfrac{2}{x-1}+\dfrac{4}{(x-1)^2}\right)dx=2\ln|x-1|-\dfrac{4}{x-1}+C$

7. a) $\int\dfrac{x-1}{x^2+4x+3}dx=\int\left(\dfrac{2}{x+3}-\dfrac{1}{x+1}\right)dx=$

$=2\ln|x+3|-\ln|x+1|+C$

b) $I=\int\dfrac{x+2-3}{(x+2)^2}dx=\int\left(\dfrac{1}{x+2}-\dfrac{3}{(x+2)^2}\right)dx=$

$=\ln|x+2|+3(x+2)^{-1}+C$

c) $I=\dfrac{1}{2}\int\dfrac{2x+4-6}{x^2+4x+5}dx=\dfrac{1}{2}\int\dfrac{2x+4}{x^2+4x+5}dx-$

$-\int\dfrac{3}{1+(x+2)^2}dx=\dfrac{1}{2}\ln(x^2+4x+5)-$

$-3\operatorname{arctg}(x+2)+C$

8. a) $I=\int\left(2x-\dfrac{2x}{x^2+1}\right)dx=x^2-\ln(x^2+1)+C$

b) $I=\int\dfrac{1}{4+(x-1)^2}dx=\dfrac{1}{2}\operatorname{arctg}\dfrac{x-1}{2}+C$

c) $I=\int\left(\dfrac{3}{x-1}+\dfrac{4}{x^2+1}\right)dx=$

$=3\ln|x-1|+4\operatorname{arctg}x+C$

9. a) $I=\int\left[(1+\operatorname{tg}^2x)-1\right]dx=\operatorname{tg}x-x+C$

b) $I=\int\dfrac{(1+\operatorname{sen}x)^2}{1-\operatorname{sen}^2x}dx=\int\left(\dfrac{1}{\cos^2x}+\dfrac{2\operatorname{sen}x}{\cos^2x}+\right.$

$\left.+\dfrac{1-\cos^2x}{\cos^2x}\right)dx=2\operatorname{tg}x+2\sec x-x+C$

c) $I=\int\operatorname{sen}^{-2}x\cos x\,dx=-\operatorname{cosec}x+C$

10. a) Si $t=\operatorname{tg}\dfrac{x}{2}\Rightarrow dx=\dfrac{2dt}{1+t^2}$ y $\cos x=\dfrac{1-t^2}{1+t^2}$

$I=\int\dfrac{1}{1+\cos x}dx=\int\dfrac{1+t^2}{2}\cdot\dfrac{2dt}{1+t^2}=t+C=\operatorname{tg}\left(\dfrac{x}{2}\right)+C$

b) $I=\dfrac{1}{4}\int(2\operatorname{sen}x\cos x)^2dx=\dfrac{1}{4}\int\operatorname{sen}^2 2x\,dx=$

$=\dfrac{1}{4}\int\dfrac{1-\cos 4x}{2}dx=\dfrac{1}{8}\left(x-\dfrac{\operatorname{sen}4x}{4}\right)+C$

c) $I=\int\dfrac{1+e^x-e^x}{1+e^x}dx==\int\left(1-\dfrac{e^x}{1+e^x}\right)dx=$

$=x-\ln\left(1+e^x\right)+C$

Soluciones propuesta B

1. $F(x)=\int(\text{sen}\,x+\cos x)\,dx=-\cos x+\text{sen}\,x+C$

$F\left(\frac{\pi}{2}\right)=-\cos\frac{\pi}{2}+\text{sen}\frac{\pi}{2}+C=1+C=-2 \Rightarrow$

$C=-3\Rightarrow F(x)=-\cos x+\text{sen}\,x-3$

2. La función es $f(x)=2x^3-2x^2+5x+K$ porque su derivada es $f'(x)=6x^2-4x+5$. Para hallar la constante *K*, se impone $f(1)=0 \Rightarrow$ $\Rightarrow K=-5$.

$f(x)=2x^3-2x^2+5x-5 \Rightarrow f(0)=-5$

3. a) De la gráfica: $v_0=v(0)=4$ m/s

b) De la gráfica: $v(t)=2t+4$ m/s

c) $s=\int v\,dt=\int(2t+4)\,dt=t^2+4t+K$

$s(4)=50\Rightarrow 16+16+K=50\Rightarrow K=18$

$s(t)=t^2+4t+18$ m

d) $s_0=s(0)=18$ m

e) $a=v'(t)=2$ m/s^2

4. a) $I=\int(10x+x^{\frac{1}{2}})\,dx=5x^2+\dfrac{2\sqrt{x^3}}{3}+K$

b) $I=\int(x^2+2x^{\frac{1}{3}})\,dx=\dfrac{x^3}{3}+\dfrac{3\sqrt[3]{x^4}}{4}+K$

c) $I=\ln|\text{tg}\,x|+K$

d) $I=\dfrac{1}{5}(e^x+2)^5+K$

e) $I=\dfrac{5}{2}(\ln x)^2+K$

f) $I=\int\left(\dfrac{2x}{x^2+1}+\dfrac{5}{x^2+1}\right)dx=$

$=\ln(x^2+1)+5\,\text{arctg}\,x+K$

5. a) $I=x\,\text{arctg}\,2x-\int\dfrac{2x}{1+4x^2}dx=$

$=x\,\text{arctg}\,2x-\dfrac{1}{4}\ln(1+4x^2)+C$

b) $I=(x+1)e^x-\int e^x dx=$

$=(x+1)e^x-e^x+C=xe^x+C$

c) $I=x\ln x^2-\int 2\,dx=x\ln x^2-2x+C$

6. a) $\int\dfrac{x+5}{x^2+x-2}dx=2\int\dfrac{dx}{x-1}-\int\dfrac{dx}{x+2}=$

$=2\ln|x-1|-\ln|x+2|+C=\ln\left|\dfrac{(x-1)^2}{x+2}\right|+C$

b) $I=\int\left(2-\dfrac{7}{x+4}\right)dx=2x-7\ln|x+4|+C$

c) $I=\int\dfrac{x+1+1}{(x+1)^2}dx=\int\dfrac{1}{x+1}dx+\int\dfrac{1}{(x+1)^2}dx=$

$=\ln|x+1|-\dfrac{1}{x+1}+C$

7. a) $I=-\dfrac{1}{2}\int\dfrac{1}{x-2}dx+\dfrac{3}{2}\int\dfrac{1}{x-4}dx=$

$=-\dfrac{1}{2}\ln|x-2|+\dfrac{3}{2}\ln|x-4|+C$

b) $I=\int\dfrac{x-3+2}{(x-3)^2}dx=\int\left(\dfrac{1}{x-3}+\dfrac{2}{(x-3)^2}\right)dx=$

$=\ln|x-3|-\dfrac{2}{x-3}+C$

c) $I=\dfrac{1}{2}\int\dfrac{2x-6+4}{x^2-6x+10}dx=$

$=\dfrac{1}{2}\int\dfrac{2x-6}{x^2-6x+10}dx+\int\dfrac{2}{1+(x-3)^2}dx=$

$=\dfrac{1}{2}\ln(x^2-6x+10)+2\,\text{arctg}(x-3)+C$

8. a) $I=\int\left(2+\dfrac{-8}{4+x^2}\right)dx=2x-4\,\text{arctg}\left(\dfrac{x}{2}\right)+C$

b) $I=\dfrac{1}{2}\int\dfrac{2x-2+2}{x^2-2x+5}dx=$

$=\dfrac{1}{2}\int\dfrac{2x-2}{x^2-2x+5}dx+\int\dfrac{1}{4+(x-1)^2}dx=$

$=\dfrac{1}{2}\ln(x^2-2x+5)+\dfrac{1}{2}\text{arctg}\dfrac{x-1}{2}+C$

c) $I=\int\left(\dfrac{3}{x-1}+\dfrac{x}{x^2+1}\right)dx=$

$=3\ln|x-1|+\dfrac{1}{2}\ln(x^2+1)+C$

9. a) $I=\int\dfrac{\text{sen}(5x-5)+\text{sen}(x+5)}{2}dx=$

$=-\dfrac{\cos(5x-5)}{10}-\dfrac{\cos(x+5)}{2}+C$

b) $I=\int\dfrac{(1+\cos x)^2}{1-\cos^2 x}dx=\int\left(\dfrac{1}{\text{sen}^2 x}+\dfrac{2\cos x}{\text{sen}^2 x}+\right.$

$\left.+\dfrac{1-\text{sen}^2 x}{\text{sen}^2 x}\right)=-2(\text{cotg}\,x+\text{cosec}\,x+x)+C$

c) $I=\int\dfrac{\cos x-\cos x\,\text{sen}^2 x}{\text{sen}^5 x}dx=\int\left(\dfrac{\cos x}{\text{sen}^5 x}-\dfrac{\cos x}{\text{sen}^3 x}\right)dx=$

$=-\dfrac{\text{cosec}^4 x}{4}+\dfrac{\text{cosec}^2 x}{2}+C$

10. a) Si $t=\text{tg}\dfrac{x}{2}\Rightarrow dx=\dfrac{2dt}{1+t^2}$ y $\text{sen}\,x=\dfrac{2t}{1+t^2}$

$I=\int\dfrac{1+t^2}{(1+t)^2}\cdot\dfrac{2dt}{1+t^2}=\dfrac{-2}{1+t}+C=\dfrac{-2}{1+\text{tg}(x/2)}+C$

b) Haciendo $3^x=t$, $3^x\ln 3\,dx=dt$,

$I=\dfrac{1}{2\ln 3}\ln\left|\dfrac{t-1}{t+1}\right|+C=\dfrac{1}{2\ln 3}\ln\left|\dfrac{3^x-1}{3^x+1}\right|+C$

c) Si $2x+1=t^2\Rightarrow 2dx=2tdt$

$I=\dfrac{t^3(3t^2-5)}{30}+C=\dfrac{3x-1}{15}(2x+1)^{\frac{3}{2}}+C$