Soluciones propuesta A

1. a) $\begin{vmatrix} \operatorname{sen}\alpha & \cos\alpha \\ -\cos\alpha & \operatorname{sen}\alpha \end{vmatrix} = \operatorname{sen}^2\alpha + \cos^2\alpha = 1$

b) $\begin{vmatrix} x+a & x-a \\ x-a & x+a \end{vmatrix} = (x+a)^2 - (x-a)^2 = 4ax$

c) $\begin{vmatrix} 3 & 0 & a-1 \\ 1 & a-1 & 1 \\ a & a-1 & 1 \end{vmatrix} = (a-1)^2(1-a) = (1-a)^3$

2. $\begin{vmatrix} 2k & 4c+2f & 3f+k \\ 2h & 4b+2e & 3e+h \\ 2g & 4a+2d & 3d+g \end{vmatrix} = \begin{vmatrix} 2k & 4c & 3f \\ 2h & 4b & 3e \\ 2g & 4a & 3d \end{vmatrix} +$

$+\begin{vmatrix} 2k & 4c & k \\ 2h & 4b & h \\ 2g & 4a & g \end{vmatrix} + \begin{vmatrix} 2k & 2f & 3f \\ 2h & 2e & 3e \\ 2g & 2d & 3d \end{vmatrix} + \begin{vmatrix} 2k & 2f & k \\ 2h & 2e & h \\ 2g & 2d & g \end{vmatrix} =$

$= 2\cdot 4\cdot 3\begin{vmatrix} k & c & f \\ h & b & e \\ g & a & d \end{vmatrix} + 0 + 0 + 0 = 24\begin{vmatrix} k & h & g \\ c & b & a \\ f & e & d \end{vmatrix} =$

$= -24\begin{vmatrix} g & h & k \\ a & b & c \\ d & e & f \end{vmatrix} = 24\begin{vmatrix} a & b & c \\ g & h & k \\ d & e & f \end{vmatrix} = -24\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & k \end{vmatrix} = -96$

3. $\begin{vmatrix} 1 & 1 & 1 \\ \operatorname{sen}\alpha & \operatorname{sen}\beta & \operatorname{sen}\gamma \\ \cos\alpha & \cos\beta & \cos\gamma \end{vmatrix} =$

$= \begin{vmatrix} \operatorname{sen}\beta & \operatorname{sen}\gamma \\ \cos\beta & \cos\gamma \end{vmatrix} - \begin{vmatrix} \operatorname{sen}\alpha & \operatorname{sen}\gamma \\ \cos\alpha & \cos\gamma \end{vmatrix} + \begin{vmatrix} \operatorname{sen}\alpha & \operatorname{sen}\beta \\ \cos\alpha & \cos\beta \end{vmatrix} =$

$= (\operatorname{sen}\beta\cos\gamma - \cos\beta\operatorname{sen}\gamma) -$

$-(\operatorname{sen}\alpha\cos\gamma - \cos\alpha\operatorname{sen}\gamma) +$

$+(\operatorname{sen}\alpha\cos\beta - \cos\alpha\operatorname{sen}\beta) =$

$= \operatorname{sen}(\beta-\gamma) - \operatorname{sen}(\alpha-\gamma) + \operatorname{sen}(\alpha-\beta) =$

$= \operatorname{sen}(\beta-\gamma) + \operatorname{sen}(\gamma-\alpha) + \operatorname{sen}(\alpha-\beta)$

4. $\begin{vmatrix} 1 & d+e & de \\ 1 & e+c & ec \\ 1 & c+d & cd \end{vmatrix} \xrightarrow[F_2-F_1]{F_3-F_1} \begin{vmatrix} 1 & d+e & de \\ 0 & c-d & e(c-d) \\ 0 & c-e & d(c-e) \end{vmatrix} =$

$\begin{vmatrix} c-d & e(c-d) \\ c-e & d(c-e) \end{vmatrix} = (c-d)(c-e)\begin{vmatrix} 1 & e \\ 1 & d \end{vmatrix} =$

$= (c-d)(c-e)(d-e)$

5. $\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \end{vmatrix} \xrightarrow[F_4-F_1]{F_2-F_1,\ F_3-F_1} \begin{vmatrix} 1 & 1 & 1 & 1 \\ a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \end{vmatrix} =$

$= -1\cdot\begin{vmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{vmatrix} = -abc$

6. $A = \begin{pmatrix} 1 & -2 & 3 & 0 \\ 1 & k & 5 & 2 \\ 3 & k-4 & k+9 & k \end{pmatrix}$

$\begin{vmatrix} 3 & 0 \\ 5 & 2 \end{vmatrix} = 6 \neq 0 \Rightarrow \operatorname{rg}(A) \geq 2$

$\begin{vmatrix} 1 & 3 & 0 \\ 1 & 5 & 2 \\ 3 & k+9 & k \end{vmatrix} = 0,$

$\begin{vmatrix} -2 & 3 & 0 \\ k & 5 & 2 \\ k-4 & k+9 & k \end{vmatrix} = -3k^2 + 12 \neq 0 \Rightarrow k \neq 2,\ k \neq -2$

Si $k \notin \{-2,\ 2\} \Rightarrow \operatorname{rg}(A) = 3$

Si $k \in \{-2,\ 2\} \Rightarrow \operatorname{rg}(A) = 2$

7. $\begin{vmatrix} 1 & 0 & -1 \\ 0 & m & 3 \\ 4 & 1 & -m \end{vmatrix} = -m^2 + 4m - 3 = 0 \Rightarrow \begin{cases} m = 1 \\ m = 3 \end{cases}$

Si $m \notin \{1,\ 3\} \Rightarrow |A| \neq 0 \Rightarrow A$ tiene inversa.

Si $m = 0 \Rightarrow |A| = -3$

$A^{-1} = \begin{pmatrix} 1 & \frac{1}{3} & 0 \\ -4 & -\frac{4}{3} & 1 \\ 0 & \frac{1}{3} & 0 \end{pmatrix}$

8. $\begin{vmatrix} 1 & x & x^2 & x^3 \\ 3 & 2x+1 & x^2+2x & 3x^2 \\ 3 & x+2 & 2x+1 & 3x \\ 1 & 1 & 1 & 1 \end{vmatrix} \rightarrow$

$\xrightarrow[C_2-C_1]{C_4-C_3,\ C_3-C_2} \begin{vmatrix} 1 & x-1 & x^2-x & x^3-x^2 \\ 3 & 2x-2 & x^2-1 & 2x^2-2x \\ 3 & x-1 & x-1 & x-1 \\ 1 & 0 & 0 & 0 \end{vmatrix} =$

$= -(x-1)\begin{vmatrix} x-1 & x(x-1) & x^2(x-1) \\ 2(x-1) & (x+1)(x-1) & 2x(x-1) \\ 1 & 1 & 1 \end{vmatrix} =$

$= -(x-1)^3\begin{vmatrix} 1 & x & x^2 \\ 2 & x+1 & 2x \\ 1 & 1 & 1 \end{vmatrix} \rightarrow$

$\xrightarrow[C_2-C_1]{C_3-C_2} -(x-1)^3\begin{vmatrix} 1 & x-1 & x^2-x \\ 2 & x-1 & x-1 \\ 1 & 0 & 0 \end{vmatrix} =$

$= -(x-1)^5\begin{vmatrix} 1 & x \\ 1 & 1 \end{vmatrix} = (x-1)^6 = 0 \Rightarrow x = 1$

Soluciones propuesta B

1. a) $\begin{vmatrix} \operatorname{sen}\alpha & -\cos\alpha \\ \operatorname{sen}\alpha & \cos\alpha \end{vmatrix} = 2\operatorname{sen}\alpha\cos\alpha = \operatorname{sen}2\alpha$

b) $\begin{vmatrix} x+a & x-a \\ x-a & x-a \end{vmatrix} = -2a^2 + 2ax = 2a(x-a)$

c) $\begin{vmatrix} a & a & a \\ a & b & b \\ a & b & c \end{vmatrix} = a^2b - a^2c - ab^2 + abc$

2. $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & k \end{vmatrix} = -\begin{vmatrix} -a & -b & -c \\ -d & -e & -f \\ -g & -h & -k \end{vmatrix} =$

$= \begin{vmatrix} -d & -e & -f \\ -a & -b & -c \\ -g & -h & -k \end{vmatrix} = 100$

3. $\begin{vmatrix} 1 & 1 & 1 \\ e^{-\pi} & e^{\pi}+e^{-\pi} & e^{\pi}+e^{-\pi} \\ e^{-\pi} & e^{\pi}+e^{-\pi} & e^{\pi}+2e^{-\pi} \end{vmatrix} =$

$\overset{F_3-F_2}{=} \begin{vmatrix} 1 & 1 & 1 \\ e^{-\pi} & e^{\pi}+e^{-\pi} & e^{\pi}+e^{-\pi} \\ 0 & 0 & e^{-\pi} \end{vmatrix} =$

$= e^{-\pi}\begin{vmatrix} 1 & 1 \\ e^{-\pi} & e^{\pi}+e^{-\pi} \end{vmatrix} =$

$= e^{-\pi}\left(e^{\pi}+e^{-\pi}-e^{-\pi}\right) =$

$= e^{-\pi}e^{\pi} = e^0 = 1$

4. a)
$\begin{vmatrix} 3k+1 & k & k \\ 6k+2 & 2k+1 & 2k \\ 3k+1 & k & k+1 \end{vmatrix} = (3k+1)\begin{vmatrix} 1 & k & k \\ 2 & 2k+1 & 2k \\ 1 & k & k+1 \end{vmatrix}$

$\xrightarrow[F_3-F_1]{F_2-2F_1} (3k+1)\begin{vmatrix} 1 & k & k \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 3k+1$

$|A| = 0 \Rightarrow k = -\dfrac{1}{3}$

b) Existe $(A(k))^{-1}$ si $k \neq -\dfrac{1}{3}$.

$A(0) = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \Rightarrow (A(0))^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$

5. $\begin{vmatrix} x & x+1 & x+2 \\ x & x+3 & x+4 \\ x & x+5 & x+6 \end{vmatrix} = \begin{vmatrix} x & 1 & 2 \\ x & 3 & 4 \\ x & 5 & 6 \end{vmatrix} =$

$= \begin{vmatrix} x & 1 & 2 \\ 0 & 2 & 2 \\ 0 & 4 & 4 \end{vmatrix} = x\begin{vmatrix} 2 & 2 \\ 4 & 4 \end{vmatrix} = 0$

6. $|A| = \begin{vmatrix} 1 & 1 & 2 & 1 \\ 3 & -1 & 2 & m \\ m+2 & -2 & 0 & -1 \\ 1 & 1 & -1 & 1 \end{vmatrix} =$

$\overset{\substack{C_1+C_3 \\ C_2+C_3 \\ C_4+C_3}}{=} \begin{vmatrix} 3 & 3 & 2 & 3 \\ 5 & 1 & 2 & m+2 \\ m+2 & -2 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{vmatrix} =$

$= 3\begin{vmatrix} 1 & 1 & 1 \\ 5 & 1 & m+2 \\ m+2 & -2 & -1 \end{vmatrix} \overset{\substack{C_1-C_2 \\ C_3-C_2}}{=} 3\begin{vmatrix} 0 & 1 & 0 \\ 4 & 1 & m+1 \\ m+4 & -2 & 1 \end{vmatrix} =$

$= -3\begin{vmatrix} 4 & m+1 \\ m+4 & 1 \end{vmatrix} = 3m(m+5)$

Si $m \notin \{0,-5\} \Rightarrow |A| \neq 0 \Rightarrow \operatorname{rg}(A) = 4$

Si $m = 0$, $A = \begin{pmatrix} 1 & 1 & 2 & 1 \\ 3 & -1 & 2 & 0 \\ 2 & -2 & 0 & -1 \\ 1 & 1 & -1 & 1 \end{pmatrix}$

$\begin{vmatrix} 2 & 1 \\ 2 & 0 \end{vmatrix} = -2 \neq 0,\ \begin{vmatrix} 1 & 2 & 1 \\ -1 & 2 & 0 \\ 1 & -1 & 1 \end{vmatrix} = 3 \neq 0 \Rightarrow \operatorname{rg}(A) = 3$

Si $m = -5$, $A = \begin{pmatrix} 1 & 1 & 2 & 1 \\ 3 & -1 & 2 & -5 \\ -3 & -2 & 0 & -1 \\ 1 & 1 & -1 & 1 \end{pmatrix}$

$\begin{vmatrix} 0 & -1 \\ -1 & 1 \end{vmatrix} = -1 \neq 0,\ \begin{vmatrix} -1 & 2 & -5 \\ -2 & 0 & -1 \\ 1 & -1 & 1 \end{vmatrix} = -7 \neq 0 \Rightarrow \operatorname{rg}(A) = 3$

7. a) $|A| = \begin{vmatrix} k-1 & 1 & -1 \\ 0 & k-2 & 1 \\ k & 0 & 2 \end{vmatrix} = (k-1)(3k-4)$

$|A| = 0 \Rightarrow \begin{cases} k = 1 \\ k = \dfrac{4}{3} \end{cases}$. Existe A^{-1} si $k \notin \left\{1, \dfrac{4}{3}\right\}$

b) $k = 2 \Rightarrow A^{-1} = \begin{pmatrix} 0 & -1 & \frac{1}{2} \\ 1 & 2 & -\frac{1}{2} \\ 0 & 1 & 0 \end{pmatrix}$

8. $\begin{vmatrix} x & 1 & 8 & 1 \\ 1 & x & 1 & 8 \\ 8 & 1 & x & 1 \\ 1 & 8 & 1 & x \end{vmatrix} \overset{C_1+C_2+C_3+C_4}{=} (x+10)\begin{vmatrix} 1 & 1 & 8 & 1 \\ 1 & x & 1 & 8 \\ 1 & 1 & x & 1 \\ 1 & 8 & 1 & x \end{vmatrix}$

$= (x+10)(x-8)^2(x+6) = 0$

Soluciones: $x = -10,\ x = -6,\ x = 8$ (doble)