Soluciones propuesta A

1. $TVM\ f[0,2]=\dfrac{f(2)-f(0)}{2-0}=\dfrac{0-(-4)}{2}=2$

$TVM\ f[a,a+h]=\dfrac{(a+h)^2-4-(a^2-4)}{h}=h+2a$

2. a) $TVI(-1)=\lim\limits_{h\to 0}\dfrac{(-1+h)^3+1}{h}=\lim\limits_{h\to 0}(h^2-3h+3)=3$

b) $TVI(-2)=\lim\limits_{h\to 0}\dfrac{\frac{h-2}{h-1}-2}{h}=\lim\limits_{h\to 0}\dfrac{-1}{h-1}=1$

c) $TVI(2)=\lim\limits_{h\to 0}\dfrac{\sqrt{4+h}-2}{h}=\lim\limits_{h\to 0}\dfrac{1}{\sqrt{4+h}+2}=\dfrac{1}{4}$

3. a) $\operatorname{tg}\alpha=m=f'(1)=3\ \Rightarrow\ \alpha=71º\ 33'\ 54''$

Ecuación de la tangente $y-1=3(x-1)$

b) $m=f'(-1)=\dfrac{-2}{(-1+3)^2}=-\dfrac{1}{2}\Rightarrow\alpha=153º\ 26'\ 6''$

Ecuación de la tangente: $y-1=-\dfrac{1}{2}(x+1)$

4. a) $(f+g)'(-1)=f'(-1)+g'(-1)=-1$

b) $(fg)'(2)=f'(2)g(2)+f(2)g'(2)=$
$=4\ (-1)+3\cdot 0=-4$

c) $\left(\dfrac{f}{g}\right)'(2)=\dfrac{f'(2)g(2)-f(2)g'(2)}{[g(2)]^2}=-4$

d) $(f\circ g)'(2)=f'(g(2))g'(2)=f'(-1)g'(2)=0$

e) $(g\circ f)'(2)=g'(f(2))f'(2)=g'(-1)f'(2)=-12$

5. Para que sea continua en $x=0$,

$\lim\limits_{x\to 0^-}f(x)=\lim\limits_{x\to 0^+}f(x)=f(0)\Rightarrow 1=b$

Para que sea derivable en $x = 0$, las derivadas laterales deben coincidir:

$f'(0^-)=f'(0^+)\Rightarrow 0=a\ \Rightarrow$

$\Rightarrow f(x)=\begin{cases}x^3+1 & \text{si } x\le 0\\ 1 & \text{si } x>0\end{cases}$

6. $f'(x)=\dfrac{-1}{2x^2},\ g'(x)=\dfrac{1}{2\sqrt{x+1}},\ h'(x)=2x$

a) $(g\circ h)'(2)=g'[h(2)]h'(2)=\dfrac{1}{2\sqrt{5}}\cdot 4=\dfrac{2}{\sqrt{5}}$

b) $(h\circ g\circ f)'(1)=h'[g(f(1))]\ g'(f(1))f'(1)=-\dfrac{1}{2}$

c) $(f\circ h\circ g)'(4)=f'[h(g(4))]\ h'(g(4))\cdot g'(4)=-\dfrac{1}{50}$

d) $(g\circ f\circ h)'(x)=\dfrac{-\sqrt{2}}{2x^2\sqrt{1+2x^2}}$

7. $f(c)=-9\Rightarrow c^3+c-11=-9\Rightarrow c=1$

$f'(x)=3x^2+1\Rightarrow f'(1)=4$

$\left(f\circ f^{-1}\right)(x)=x$. Derivando:

$f'\left(f^{-1}(x)\right)\cdot\left(f^{-1}\right)'(x)=1\Rightarrow$

$\Rightarrow\left(f^{-1}\right)(-9)=\dfrac{1}{f'\left(f^{-1}(-9)\right)}=\dfrac{1}{f'(1)}=\dfrac{1}{4}$

8. a) $a'(x)=\operatorname{sen}2x$ d) $d'(x)=\dfrac{-2x}{\sqrt{1-x^4}}$

b) $b'(x)=\dfrac{2}{\sqrt{1-4x^2}}$ e) $e'(x)=2\sec^2(2x)$

c) $c'(x)=\dfrac{-2\arccos x}{\sqrt{1-x^2}}$ f) $f'(x)=\dfrac{-2}{x^2}\sec^2\left(\dfrac{2}{x}\right)$

9. a) $a'(x)=-\dfrac{\operatorname{tg}x}{x}$ e) $e'(x)=e^x(x+1)$

b) $b'(x)=\dfrac{\cos x}{x}-\ln x\cdot\operatorname{sen}x$ f) $f'(x)=1$

c) $c'(x)=-(1+\ln x)\operatorname{sen}(x\ln x)$

d) $d'(x)=2e^{2x}$ g) $g'(x)=2^x\left(\ln 2\cdot x^2+2x\right)$

h) $h(x)=\ln\sqrt{\dfrac{1}{x}}=-\dfrac{1}{2}\ln x\Rightarrow h'(x)=-\dfrac{1}{2x}$

10. a) $v_m=\dfrac{s(4)-s(1)}{4-1}=\dfrac{47-2}{3}=15\ \text{ms}^{-1}$

b) $v_i=s'(2)=6\cdot 2=12\ \text{ms}^{-1}$

11. $y=\ln x\Rightarrow dy=\dfrac{dx}{x},\ f(x+dx)\approx f(x)+dy$

$\ln(54)\approx\ln(50)+\dfrac{1}{50}\cdot 4=3{,}912+0{,}08=3{,}992$

$Er=\dfrac{|3{,}992-3{,}98898|}{3{,}98898}=0{,}0756\%$

$\ln(46)\approx\ln(50)+\dfrac{1}{50}(-4)=3{,}912-0{,}08=3{,}832$

$Er=\dfrac{|3{,}832-3{,}82864|}{3{,}82864}\cdot 100=0{,}0877\%$

$\ln(40)\approx\ln(50)+\dfrac{1}{50}\cdot(-10)=3{,}912-0{,}2=3{,}712$

$Er=\dfrac{|3{,}872-3{,}68888|}{3{,}68888}\cdot 100=4{,}96\%$

cuanto más lejos de $x = 50$, mayor es el error.

12. $V=\dfrac{4}{3}\pi r^3,\ dV=4\pi r^2dr$

$\Delta V\approx dV=4\pi 2^2\cdot 0{,}03=1{,}508\ \text{m}^3$

Soluciones propuesta B

1. $TVM\ f[-3,1]=\dfrac{f(1)-f(-3)}{1-(-3)}=\dfrac{1-(-27)}{4}=7$

$TVM\ f[a,a+h]=\dfrac{(a+h)^3-a^3}{h}=h^2+3ha+3a^2$

2. a) $TVI(3)=\lim\limits_{h\to 0}\dfrac{(3+h)^2-1-8}{h}=\lim\limits_{h\to 0}(h+6)=6$

b) $TVI(-2)=\lim\limits_{h\to 0}\dfrac{\dfrac{-2+h-1}{-2+h}-\dfrac{3}{2}}{h}=\lim\limits_{h\to 0}\dfrac{-1}{2(h-2)}=\dfrac{1}{4}$

c) $TVI(1)=\lim\limits_{h\to 0}\dfrac{\sqrt{1+h}-1}{h}=\lim\limits_{h\to 0}\dfrac{1}{\sqrt{1+h}+1}=\dfrac{1}{2}$

3. a) $m=f'(2)=3=\operatorname{tg}\alpha\Rightarrow\alpha=71º\ 33'\ 54''$

Ecuación de la normal: $y-8=-\dfrac{1}{3}(x-2)$

b) $m=f'(0)=\dfrac{0}{\sqrt{0^2+1}}=0=\operatorname{tg}\alpha=0\Rightarrow\alpha=0º$

La tangente es horizontal y la normal es la recta vertical x = 0.

4. a) $(f+g)'(1)=f'(1)+g'(1)=2+\left(-\dfrac{3}{4}\right)=\dfrac{5}{4}$

b) $(fg)'(2)=f'(2)g(2)+f(2)g'(2)=\dfrac{11}{3}$

c) $\left(\dfrac{f}{g}\right)'(2)=\dfrac{f'(2)g(2)-f(2)g'(2)}{[g(2)]^2}=\dfrac{13}{3}$

d) $(f\circ g)'(2)=f'(g(2))\cdot g'(2)=f'(1)\,g'(2)=-\dfrac{2}{3}$

e) $(g\circ f)'(2)=g'(f(2))f'(2)=g'(1)f'(2)=-3$

5. Si $x\neq 1$, f es derivable al estar definida por polinomios. Para que sea continua en x = 1:

$\lim\limits_{x\to 1^-}f(x)=\lim\limits_{x\to 1^+}f(x)=f(1)\Rightarrow$

$\Rightarrow a+b-1=2b-2\Rightarrow a=b-1$

Para que sea derivable en x = 1, las derivadas laterales deben coincidir:

$f'(1^-)=f'(1^+)\Rightarrow 2a+b=2b\Rightarrow 2a-b=0$

Y de aquí: $\begin{cases}a-b=-1\\2a-b=0\end{cases}\Rightarrow a=1,\ b=2$

6. $f^{-1}(c)=\ln 4\Rightarrow c\cdot e^c=\ln 4\Rightarrow c=\ln 2$

$\left(f^{-1}\right)'(x)=e^x(1+x)\Rightarrow\left(f^{-1}\right)'(\ln 2)=2(1+\ln 2)$

$\left(f^{-1}\circ f\right)(x)=x$. Derivando la función compuesta:

$\left(f^{-1}\right)'(f(x))f'(x)=1\Rightarrow f'(\ln 4)=$

$=\dfrac{1}{\left(f^{-1}\right)'(f(\ln 4))}=\dfrac{1}{\left(f^{-1}\right)'(\ln 2)}=\dfrac{1}{2(1+\ln 2)}$

Ec. tangente : $y-\ln 2=\dfrac{1}{2(1+\ln 2)}(x-\ln 4)$

7. a) $a'(x)=-\operatorname{sen}2x$ c) $c'(x)=2\cos(2x)$

b) $b'(x)=\dfrac{2x}{\sqrt{1-x^4}}$ d) $d'(x)=2\operatorname{tg}x\ \sec^2 x$

e) $e'(x)=-\dfrac{1}{x^2}\cdot\dfrac{1}{1+\left(\dfrac{1}{x}\right)^2}=\dfrac{-1}{x^2+1}$

f) $f'(x)=\dfrac{-2}{(1+x)^2}\cdot\dfrac{1}{1+\left(\dfrac{1-x}{1+x}\right)^2}=\dfrac{-1}{1+x^2}$

8. a) $a'(x)=2\ln 5\cdot 5^{2x}$ e) $e'(x)=\dfrac{1}{2\sqrt{x}\left(1+x^2\right)}$

b) $b'(x)=x\cdot e^x$ f) $f'(x)=\dfrac{1}{2x}$

c) $c'(x)=1$ g) $g'(x)=\dfrac{-2x+3}{\left(x^2-3x\right)}\cdot\dfrac{1}{\ln 10}$

d) $d'(x)=3^x x^2\left(x\ln 3+3\right)$

h) $h(x)=\dfrac{1}{2}\left[\ln(1-\operatorname{sen}x)-\ln(1+\operatorname{sen}x)\right]$

$h'(x)=\dfrac{1}{2}\left[\dfrac{-\cos x}{1-\operatorname{sen}x}-\dfrac{\cos x}{1+\operatorname{sen}x}\right]=-\dfrac{1}{\cos x}$

9. $3x^2+6xy+3x^2y'+2yy'=0\Rightarrow y'=\dfrac{-3x^2-6xy}{3x^2+2y}$

$f'(-2,-2)=-\dfrac{9}{2}$. Ecuación: $y+2=-\dfrac{9}{2}(x+2)$

10. a) $\ln f(x)=\dfrac{1}{x}\ln(5-4x)\Rightarrow$

$\dfrac{f'(x)}{f(x)}=-\dfrac{1}{x^2}\ln(5-4x)+\dfrac{-4}{x(5-4x)}$

$f'(x)=\sqrt[x]{5-4x}\left[-\dfrac{1}{x^2}\ln(5-4x)+\dfrac{-4}{x(5-4x)}\right]$

b) $\ln g(x)=2x\cdot\ln(3x+1)\Rightarrow$

$\Rightarrow g'(x)=(3x+1)^{2x}\left[2\ln(3x+1)+\dfrac{6x}{3x+1}\right]$

11. a) $dy=\dfrac{-39}{(x+5)^2}dx$

b) $dx=A\omega\cos(\omega t+\phi_0)dt$

12. $f(3+dx)=f(3)+\Delta y\approx f(3)+dy$

$dy=(5x^4-12x^2-3)dx$; $dy(3)=294\,dx$

$f(3{,}001)=3{,}001^5-4\cdot 3{,}001^3-3\cdot 3{,}001\approx$

$\approx f(3)+dy=126+294\cdot 0{,}001=126{,}294$